On homotopy theory of diffeological spaces
Kazuhisa Shimakawa (Okayama University)

Let \((X, A)\) be a pair of diffeological spaces. Then its homotopy groups \(\pi_n(X, A, x_0)\) can be “formally” defined as the sets of homotopy classes of smooth maps from \((\tilde{I}^n, \partial\tilde{I}^n, \tilde{J}^{n-1})\) to \((X, A, x_0)\). Here \(\tilde{I}\) is the unit interval \([0, 1]\) endowed with the quotient diffeology given by the smashing function \(\lambda: \mathbb{R} \to [0, 1]\), and \(\tilde{J}^{n-1}\) is the subset \((\partial\tilde{I}^{n-1} \times \tilde{I}) \cup (\tilde{I}^{n-1} \times \partial\tilde{I})\) of \(\tilde{I}^n\). In this talk, I shall give an explicit construction providing a proof of the following

**Proposition.** For \(n \geq 1\), \(\tilde{J}^{n-1}\) is a smooth deformation retract of \(\tilde{I}^n\).

Just as in the category of topological spaces, the proposition above plays a key role to establish “homotopy theory” in the category \(\text{Diff}\) of diffeological spaces. In terms of homotopical algebra, this can be summarized as follows:

Let \(I\) and \(J\) be the sets of inclusions \(\partial\tilde{I}^n \to \tilde{I}^n\) and \(\tilde{J}^{n-1} \to \tilde{I}^n\), respectively. Let \(W\) be the class of weak homotopy equivalences in \(\text{Diff}\), i.e. the class of smooth maps \(f: X \to Y\) between diffeological spaces inducing isomorphisms \(\pi_n(X, x_0) \to \pi_n(Y, f(x_0))\) for any \(n \geq 0\) and \(x_0 \in X\). Then we have

**Theorem** (Haraguchi and S). *There is a finitely generated model category structure on \(\text{Diff}\) with \(I\) as the set of generating cofibrations, \(J\) as the set of generating acyclic cofibrations, and with \(W\) as the class of weak equivalences.*